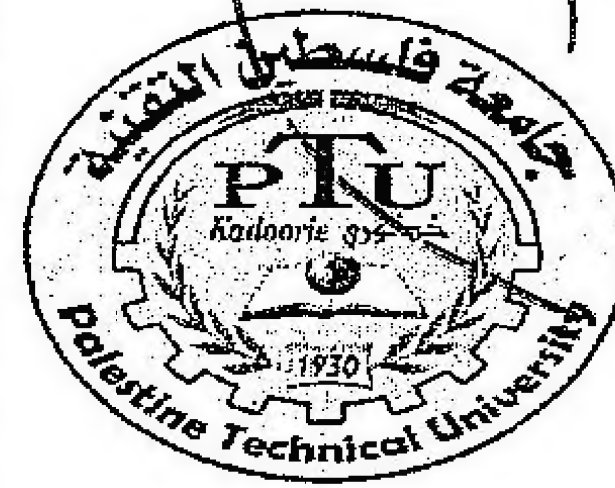


ثانية = فنية (1)
Second

التخصص والمستوى:	السلطة الوطنية الفلسطينية
اسم المساق: Linear and diffe	جامعة فلسطين التقنية "خضوري"
تاريخ الإمتحان: 2010/12/27	الامتحان الثاني للعام الدراسي 2011/2010
الإمتحان: زمن ساعة	الفصل الدراسي الأول



اسم المحاضر: د. خالد زلال

اسم الطالب:
 الشعبة:

Q1. Mark each statement by true or false.

(25 points.)

- 1) (☒ T) If $S = \{x_1, x_2, x_3\}$ are linearly independent vectors in R^3 , then S form a basis for R^3 .
- 2) (☒ F) An $n \times n$ matrix A is singular if the nullity $(A) = 0$.
- 3) (☒ F) If $L: V \rightarrow W$ is a linear transformation, then kernel (L) is a subspace of the range of (L) .
- 4) (☒ T) If A is an $n \times n$ matrix and $\lambda_1, \dots, \lambda_n$ are the eigenvalues of A with corresponding eigenvector X_1, \dots, X_n respectively, and if X_1, \dots, X_n are linearly independent then $\lambda_1, \dots, \lambda_n$ are distinct.
- 5) (☒ F) The product of the eigenvalues for an $n \times n$ matrix (A) is equal to trace of (A) .
- 6) (☒ T) If $\text{rank}[A|b] = \text{rank}(A)$ then the system $Ax = b$ is consistent.
- 7) (☒ F) The function $f(x, y) = \sqrt{x^2 - y^2}$ is continuous in the entire plane.
- 8) (☒ T) A basis for a vector space V is the smallest spanning and the largest linearly independent subset of V .
- 9) (☒ T) An $n \times n$ matrix A is singular iff one of its eigenvalues is zero.
- 10) (☒ T) If A is a 3×4 matrix, then the row vector of A are linearly dependent.

تم الرفع
بواسطة
م. معن
أبو عيسى

Q2) a. $z = \frac{xy + x \cos y}{x^2 y}$ find $\frac{\partial z}{\partial x \partial y}$ at $(x, y) = (1, \frac{\pi}{2})$

F_{yx}

(10 points)

~~$F_y = (xy + x \cos y)$~~

$F_y = \frac{xy + x \cos y}{x^2 y} \Rightarrow F_y = \frac{(xy) \cdot x^2 - x^2 y (x)}{(x^2 y)^2}$

$= \frac{x^2 y x^2 - x^3 y x}{x^4 y^2} = \frac{x(x^3 y - x^3 y)}{x^4 y^2}$

$\Rightarrow F_y = \frac{x^3 y - x^3 y}{x^3 y^2} = 0$

$$z = \frac{xy + x \cos y}{x^2 y}$$

find $\frac{\partial z}{\partial x \partial y}$ at $x=1, y=\frac{\pi}{2}$

$$F_y = \frac{x^2 y \cdot x - xy \cdot 2xy}{(x^2 y)^2}$$

$$= \frac{x^3 y - 2x^2 y^2}{x^4 y^2}$$

(1)

$$= \frac{x^2(xy - 2y^2)}{x^2(x^2 y^2)}$$

$$\Rightarrow f_y = \frac{xy - 2y^2}{x^2 y^2}$$

$$\Rightarrow f_y = \frac{y(x - 2y)}{y(x^2 y)} \Rightarrow f_y = \frac{x - 2y}{x^2 y}$$

$$f_{yx} =$$

$$f_y = \frac{x - 2y}{x^2 y}$$

$$f_{yx} = \frac{(x^2 y \cdot 1) - (x - 2y) \cdot 2xy}{(x^2 y)^2}$$

$$\Rightarrow f_{yx} = \frac{x^2 y - (2x^2 y - 4xy^2)}{(x^2 y)^2}$$

$$\Rightarrow f_{yx} = \frac{x y (x - (2x - 4y))}{x y (x^3 y)}$$

$$f_{yx} = \frac{x - 2x + 4y}{x^3 y}$$

$$= \frac{1 - 2(1) + 4(\frac{\pi}{2})}{1 \cdot \frac{\pi}{2}}$$

$$= \frac{-1 + 2\pi}{\frac{\pi}{2}}$$

$$= \frac{-2 + 4\pi}{11}$$

b. 1) Show that $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $L(\vec{x}) = (x_1, 0, x_1 + x_2)$ is a linear

transformation.

(5 points)

$$\begin{aligned} \alpha L(\vec{x}) + \beta L(\vec{y}) &= \alpha (x_1, 0, x_1 + x_2) + \beta (y_1, 0, y_1 + y_2) \\ &= (\alpha x_1, 0, \alpha x_1 + \alpha x_2) + (\beta y_1, 0, \beta y_1 + \beta y_2) \\ &= \alpha L\vec{x} + \beta L\vec{y} \rightarrow \text{transformation} \end{aligned}$$

2. Find $\ker(L)$.

(5 points)

$$\begin{aligned} \vec{x}_1, 0, x_1 + x_2 &= (0, 0, 0) \\ \begin{pmatrix} x_1 \\ 0 \\ x_1 + x_2 \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow x_1 = -x_2 \Rightarrow \ker L \text{ spanning by } \\ &\begin{pmatrix} e_1 \\ 0 \\ -e_2 \end{pmatrix} \end{aligned}$$

3. Let S be the subspace of \mathbb{R}^3 spanned by $\{e_2, e_3\}$. Find $L(S)$.

(5 points)

$$\begin{pmatrix} 0 \\ e_2 \\ e_3 \end{pmatrix}$$

Q3. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 4 \\ 1 & 2 & 4 & -1 \end{bmatrix}$. Find

1. A basis for the row space of A:

(10 points)

$$\begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 4 \\ 1 & 2 & 4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & 1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for the row space of A is
 $\text{row}(1 \ 1 \ 3 \ 1) \Rightarrow (1 \ 1 \ 3 \ 1)$
 $\text{row}(0 \ 1 \ 1 \ -2) \Rightarrow (2 \ 1 \ 5 \ -1)$

2. A basis for the column space of A:

(5 points)

A basis for column space of A
 column that has 1 only 1 & 2

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

3. A basis for the null space of A:

(10 points)

basis for the null space of A

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 4 & 0 \\ 1 & 2 & 4 & -1 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

x_4 is free variable $x_4 = \beta$, $x_3 = \alpha$

$$x_2 = -x_3 + 2x_4 = -\alpha + 2\beta$$

$$x_1 = -x_2 - 3x_3 - x_4 = -(-\alpha + 2\beta) - 3\alpha - \beta = \alpha - 2\beta - 3\alpha - \beta = -2\alpha - 3\beta$$

$$x_1 = \alpha + 2\beta - 3\alpha - \beta$$

$$x_2 = -\alpha + 2\beta$$

$$X = \begin{pmatrix} \alpha + 2\beta - 3\alpha - \beta \\ -\alpha + 2\beta \\ \alpha \\ \beta \end{pmatrix}$$

$$= \alpha \begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \rightarrow \text{کجه}$$

4. a. Find the eigenvalues and the corresponding eigenspaces for the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \text{ Is } A \text{ diagonalizable? Why?}$$

- b. Find A^{100} . Do NOT compute X^{-1}

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$1-\lambda ((2-\lambda)(1-\lambda) - (0) + 1(0))$$

$$= 1-\lambda (2-2\lambda-\lambda+\lambda^2)$$

$$= 1-\lambda (2-3\lambda+\lambda^2)$$

$$1-\lambda (\lambda^2-3\lambda+2) = 0$$

$$\frac{\lambda^2-3\lambda+2-\lambda^3+3\lambda^2-2\lambda}{-\lambda^3+4\lambda^2-5\lambda+2}$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda=1 \quad \lambda=2$$

$$1-\lambda=0 \Rightarrow \lambda=1$$

$$\lambda_1=1, \lambda_2=1, \lambda_3=2$$

For $\lambda=2$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

for $\lambda = 1$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

~~A not~~

A not diagonalizable because ~~$\lambda = 0$~~

(b) find $A^{100} =$ ~~A^{100}~~

$$A^{100} = X \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} X^{-1}$$

when it diagonalizable

D

$$A^{100} = X D^{100} X^{-1}$$